# Year 12 Mathematics Specialist 2018

# Test Number 1:  Complex Numbers

# Resource Free

**Name:** _____  **Teacher: DDA**

**Marks:**           45

**Time Allowed:**        45 minutes

**Instructions:**  You **ARE NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

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**Question 1**                                                                                      **[4 marks]**
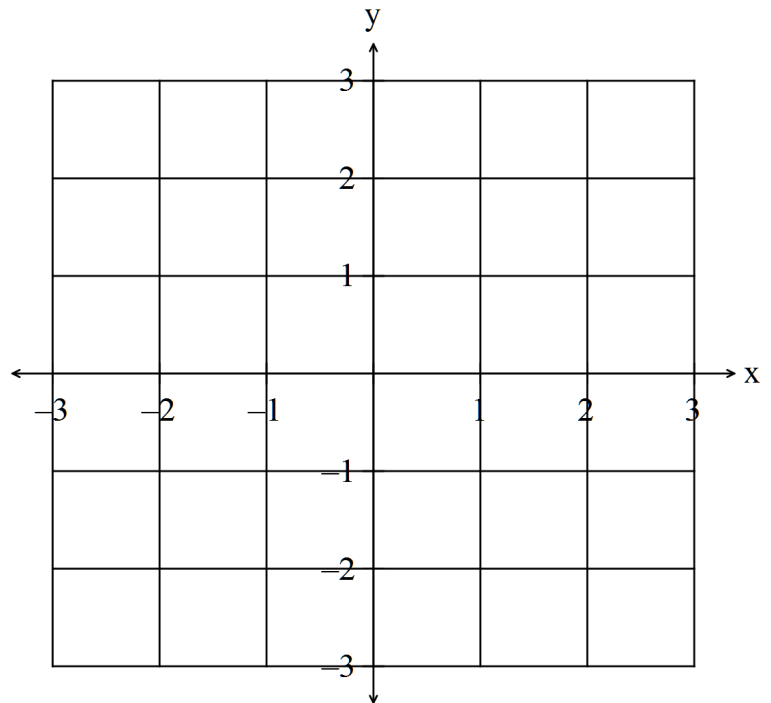
Solve the complex equation $z^4 = -16$.

**Question 2** [3 marks]

Given $P(x) = x^3 + x^2 + x - 3$ find $x$ such that $P(x) = 0$ and hence solve the equation $x^3 + x^2 + x - 3 = 0$.

**Question 3** [5,3 = 8 marks]

(a) Sketch $\left\{z:\left|x-1+iy\right|=2\left|x+i\left(y-1\right)\right|\right\}$ on the set of axes below.

(b)  If $z = \dfrac{1+i}{1-i} \times (3+3i)$ find the expression for $\bar{z}$.

**Question 4**                                                                                                    **[6,4 = 10 marks]**

(a)   Use De Moivre's theorem to express $\cos(3x)$ in terms of $\cos(x)$.

(b)    Calculate $(-1 - i)^{10}$. Give your answer in cartesian form.

**Question 5** [4 marks]

Determine the complex number $z = a + bi$, where $a, b$ are real constants with $a > 0$ such that $Im\left(\frac{1}{z^2}\right) = \frac{1}{100}$ and $Im(z) = -2Re(z)$.

Simplify the expression below.

$$\left(\frac{\sqrt{3}\,cis\,\frac{3\pi}{4}}{6\,cis\,\frac{5\pi}{6}\,cis\,\frac{2\pi}{3}}\right)^{-1}$$

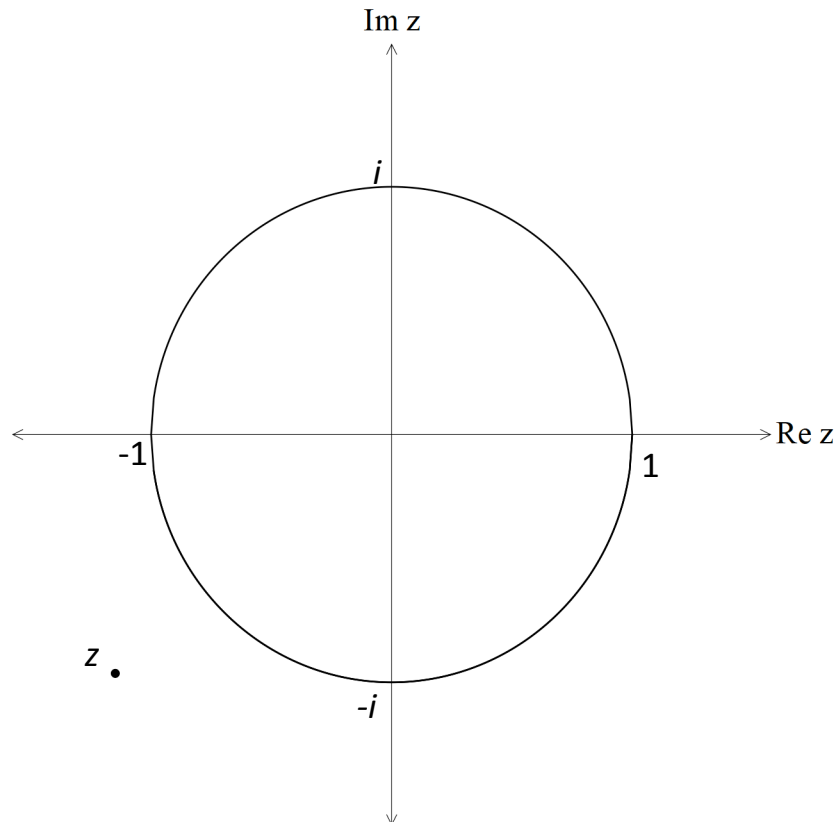**Question 7**                                                    **[2,3,2 = 7 marks]**

a) Given $z$ is a complex number with modulus $r$ and argument $\theta$, express the modulus and argument of each of the complex numbers $z_1$ and $z_2$ in terms of $r$ and $\theta$ where

i)   $z_1 = \bar{z}$.

ii) $z_2 = -z^{-1}$.

b) The diagram below shows the circle in the complex plane and the position of the complex number $z$.



Given the approximate values of $r$ and $\theta$ are 1.5 and 220° respectively, indicate the locations of he complex numbers $z_1$ and $z_2$ as defined in part (a) on the diagram above.

**Question 8**          **[3 marks]**

If $w$ is any complex cube root of unity, simplify $(1 + 4w)(1 + 4w^2)$.

**Question 9**          **[3 marks]**

Describe the locus of $z$ in the following Argand Diagram.